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APPLICATION OF LINEAR PROGRAMMING MODEL TO UNSECURED LOANS AND BAD DEBT RISK CONTROL IN BANKS

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ABSTRACT

Most banks fail as a result of mismanagement of credit risk. In this paper, the management of credit risk as it affects loan portfolio management and proactive strategy to seek out relative value opportunities are considered. An operational research technique, linear programming, is applied to the management of loan portfolio of banks. With the results obtained, using Simplex method, an answer is provided to the question of how to avoid possible occurrence of non-performing loans, bad and doubtful debts in banks when some percentage of the loans they give out are not secured.

KEYWORDS: Bad Debt, Linear Programming, Loan Portfolio, Credit Risk, Sensitivity Analysis, Simplex Method, Unsecured Loan

INTRODUCTION

Apart from fraud, one of the greatest problems facing most banks today is debts (non-performing debts or loans, bad and doubtful debts) [l]. The success of any bank in a very competitive lending environment will depend largely on the way and manner the loan portfolio of the bank is being managed. An effective way of evaluating bank's credit policies for bad debt is through the linear programming approach. Linear programming is a time-lasted-problem solving approach that enhances decision making of managers especially when certain restrictions or constraints exist which could affect the decision making process. It is a procedure for finding the maximum or minimum of a linear function where the augments are subject to linear constraints. The Simplex method, first proposed by Dantzig in 1947 [9], is one well known algorithm belonging to this class [4].Bad debt can be defined as debt that occurs when a firm believes that a debtor is unable or unwilling to pay and business will never be able to recover the money owed[3]. Bad debt has been a critical issue in the banking sector. Some of these issues arise when loan are given to customers, but due to insincerity or other reasons these loans are not paid back.

This affects the banks returns. Despite the low emphasis on loans and advances in most economies, the problem of bad debts would continually constitute a major threat to the banking industry. This paper seeks to find an optimal way of managing the loan portfolio of banks in other to maximize profit. Linear programming technique is adopted. This technique has become, nowadays, a quantitative technique most decision makers use in solving a variety of problems related to management decision, from scheduling, media selection portfolio selection, firm planning, financial planning to capital budgeting and transportation [5]. The bank loan strategy should seek to construct a broadly diversified portfolio of bank loans that maximizes return and minimizes default risk over the long run [1].

FORMULATION OF MODEL

The various notations and variables to be used in developing the model are defined as follows:

Definition of the Decision Variables

The values of x_{ij} denotes the unsecured amount to be allocated to investment type j within the loan class i.

The decision variables are shown in the following table:

Table 1: Showing the Decision Variables

S/N	Loan Classification	Investment Type	Source of Loan Finance	Decision Variables
1	Long term loan	Mortgage finance Lease Finance	Commercial Bank	x_{11} x_{12}
2	Medium term loan	Hire purchase SMS Enterprise	Commercial Bank	x_{21} x_{22}
3	Short term loan	LPO Finance Contract Finance	Commercial Bank	x_{31} x_{32}

Where

x_{11}= the percentage of unsecured long term loan to be allowed to mortgage finance within the long term scheme.

x_{12} = the percentage of unsecured long term loan to be allocated to lease finance within the long term scheme.

x_{21} = the percentage of unsecured medium term loan to be allocated to Hire purchase within the medium term loan scheme.

x_{22}= the percentage of unsecured medium term loan to be allocated to SMS Enterprise within the medium term scheme.

x_{31}= the percentage of unsecured short term loan to be allocated to LPO Finance within the short term scheme

x_{32}= the percentage of unsecured short term loan to be allocated to lease Contract Finance within the short term scheme

Table 2: Amount of Loan per Investment Type per Loan Class for 5 Years

S/N	Source of Loan	Loan Class Specification	Investment Type	Amount of Loan	Decision Variables
1	Commercial Bank	Long term loan $3,331,920,000$	Mortgage finance Lease Finance	$a = 1331920000$ $b = 2\,000000000$	x_{11} x_{12}
2	Commercial Bank	Medium term loan $2,323,440,000$	Hire Purchase SMS Enterprise	$c = 1000000000$ $d = 1323440000$	x_{21} x_{22}
3	Commercial Bank	Short term loan $3,983,040,000$	LPO Finance Contract Finance	$e = 2183040000$ $f = 1800000000$	x_{31} x_{32}

Formulation of Objective Function

The objective function to be minimized is

Minimize $Z = ax_{11} + bx_{12} + cx_{21} + dx_{22} + ex_{31} + fx_{32}$

Where a =Amount of loan for mortgage finance

b =Amount of loan forlease finance

c =Amount of loan forhire purchase

d =Amount of loan forsmall and medium enterprise finance

e =Amount of loan forLPO finance

f =Amount of loan forcontract finance

We then have:

Minimize $Z = 1{,}331{,}920{,}000x_{11} + 2{,}000{,}000{,}000x_{12} + 1{,}000{,}000{,}000x_{21} +$

$1{,}323{,}440{,}000x_{22} + 2{,}183{,}040{,}000x_{31} + 1{,}800{,}000{,}000x_{32}$

Formulation of Model Constraints

The above objective function is subjected to the following constraints:

- In line with the banks' policy to encourage small and medium scale businesses to have access to loan. At least 50% of medium term loan generated from commercial bank is unsecured.

  That is

  $x_{21} + x_{22} \geq 1{,}161{,}720{,}000$ (50% of 2,323,440,000)

- All the investment type under short term loan are at least 70% unsecured in order to encourage the small and medium scale business owners to have access to soft loans.

  $x_{31} + x_{32} \geq 2{,}788{,}128{,}000$ (70% of 3,983,040,000)

- In accordance with the policy of the bank, at least 20% of all loans should be without collateral.

  That is

  $x_{11} + x_{12} + x_{21} + x_{22} + x_{31} + x_{32} \geq 1{,}327{,}680{,}000$ (20% of 6,638,400,000)

The Model

Minimize $Z = 1{,}331{,}920{,}000x_{11} + 2{,}000{,}000{,}000x_{12} + 1{,}000{,}000{,}000x_{21} +$

$1{,}323{,}440{,}000x_{22} + 2{,}183{,}040{,}000x_{31} + 1{,}800{,}000{,}000x_{32}$

Subject to:

$x_{21} + x_{22} \geq 1{,}161{,}720{,}000$ (50% of 2,323,440,000)

$x_{31} + x_{32} \geq 2{,}788{,}128{,}000$ (70% of 3,983,040,000)

$x_{11} + x_{12} + x_{21} + x_{22} + x_{31} + x_{32} \geq 1{,}327{,}680{,}000$ (20% of 6,638,400,000)

$x_{ij} \geq 0; i = 1{,}2{,}3\ and\ j = 1{,}2$

The dual can be formulated as:

Maximize $P = 1{,}161{,}720{,}000\,x + 2{,}788{,}128{,}000\,y + 1{,}327{,}680{,}000\,z$

Subject to:

$z \leq 1{,}331{,}920{,}000$

$z \leq 2{,}000{,}000{,}000$

$x + z \leq 1{,}000{,}000{,}000$

$x + z \leq 1,323,440,000$

$y + z \leq 2,183,040,000$

$y + z \leq 1,800,000,000$

$x \geq 0, y \geq 0, z \geq 0$

MODEL SOLUTIONS

We use simplex method to solve the standard maximization problem. The initial tableau is set up and slack variables r, s, t, u, v, w are introduced in order to replace the inequalities in the constraints with equality. The dual of the model can now be written as follows:

Maximize $P = 1,161,720,000x + 2,788,128,000y + 1,327,680,000z$

Subject to:

$z + r = 1,331,920,000$

$z + s = 2,000,000,000$

$x + z + t = 1,000,000,000$

$x + z + u = 1,323,440,000$

$y + z + v = 2,183,040,000$

$y + z + w = 1,800,000,000$

$x \geq 0, y \geq 0, z \geq 0$

Forming the initial simplex tableau we have:

Tableau 3

Rows	x	y	z	r	s	t	u	v	w	RHS
R_1	0	0	1	1	0	0	0	0	0	1,331,920,000
R_2	0	0	1	0	1	0	0	0	0	2,000,000,000
R_3	1	0	1	0	0	1	0	0	0	1,000,000,000
R_4	1	0	1	0	0	0	1	0	0	1,323,440,000
R_5	0	1	1	0	0	0	0	1	0	2,183,040,000
R_6	0	1	1	0	0	0	0	0	1	1,800,000,000
R_7	1,161,720,000	2,788,128,000	1,327,680,000	0	0	0	0	0	0	0

Tableau 4

	x	w	z	r	s	t	u	v	w	RHS
r	0	0	1	1	0	0	0	0	0	1,331,920,000
s	0	0	1	0	1	0	0	0	0	2,000,000,000
t	1	0	1	0	0	1	0	0	0	1,000,000,000
u	1	0	1	0	0	0	1	0	0	1,323,440,000
v	0	1	0	0	0	0	0	1	-1	383,040,000
y	0	1	1	0	0	0	0	0	1	1,800,000,000
p	1,161,720,000	0	$-1,460,448,000$	0	0	0	0	0	$-2,788,128,000$	-50186304×10^{11}

Tableau 5

	t	w	z	r	s	t	u	v	w	RHS
r	0	0	1	1	0	0	0	0	0	1,331,920,000
s	0	0	1	0	1	0	0	0	0	2,000,000,000
x	1	0	1	0	0	1	0	0	0	1,000,000,000
u	0	0	0	0	0	-1	1	0	0	1,323,440,000
v	0	0	0	0	0	0	0	1	-1	383,040,000
y	0	1	1	0	0	0	0	0	1	1,800,000,000
p	0	0	$-2{,}622{,}168{,}000$	0	0	$-1{,}161{,}720{,}000$	0	0	$-2{,}788{,}128{,}000$	-61803504×10^{11}

RESULT ANALYSIS AND INTERPRETATION

The optimal solution to the dual problem is read from the trial table as

$$x = 1{,}000{,}000{,}000$$

$$y = 1{,}800{,}000{,}000$$

$$P = 61803504 \times 10^{11}$$

The optimal solution of the primal problem appears under the slack variables in the last row of the simplex tableau associated with the dual problem

$$x_{21} = 1161720000$$

$$y_{32} = 2788128000$$

$$z = 61803504 \times 10^{11}$$

Substituting these values into the primal objective function and the dual objective function, respectively, give the same result ($61{,}803{,}504 \times 10^{11}$) which implies the maximum value of P is equal to the minimum value of z.

The values under the slack variable columns of the trial tableau of the dual problem represent the percentage of the unsecured loan to be given out.

The value of-1161720000 in the t column suggests only 1161720000 naira unsecured medium term loan should be given for higher purchase investment type.

The value of-2788128000 in the w column means only 2788128000 naira unsecured short term loan should be given out for contract finance investment type.

The total unsecured loan that is safe to be given out in 5 years in order to control the risk of bad debt to a large extent and still achieve the goals of the bank is$61{,}803{,}504 \times 10^{11}$.

SENSITIVITY ANALYSIS OF THE MODEL PARAMETERS

Sensitivity analysis of the parameter in the model is significant because financial institutions such as banks typically lend large sums of money to borrowers over long period of time. [1]

We now determine how different values of the decision variables x_{ij} (independent variable) will impact the objective function z (a particular dependent variable) under a given set of assumptions.

Minimize

$$Z' = 1331920000x'_{11} + 2000000000x'_{12} + 1000000000x'_{21} + 1323440000x'_{22} +$$

$2183040000x'_{31} + 1800000000x'_{32}$

Subject to:

$x'_{21} + x'_{22} \geq 0.1\ \%\ of\ 2{,}323{,}440{,}000$

$x'_{31} + x'_{32} \geq 0.2\ \%\ of\ 3{,}983{,}040{,}000$

$x'_{11} + x'_{12} + x'_{21} + x'_{22} + x'_{31} + x'_{32} \geq 0.3\ \%\ 0f\ 6638400000$

$x'_{11} \geq 0, x'_{12} \geq 0, x'_{21} \geq 0, x'_{22} \geq 0, x'_{31} \geq 0, x'_{32} \geq 0$

The constraints now become:

$x'_{21} + x'_{22} \geq 2{,}323{,}440$

$x'_{31} + x'_{32} \geq 7{,}966{,}080$

$x'_{11} + x'_{12} + x'_{21} + x'_{22} + x'_{31} + x'_{32} \geq 19{,}915{,}200$

$x'_{ij} \geq 0;\ i = 1,2,3\ and\ j = I,2,3$

The dual can now be represented as follows

$P' = 2323440x' + 7966080y' + 19915200z'$

Subject to:

$z \leq 1{,}331{,}920{,}000$

$z \leq 2{,}000{,}000{,}000$

$x + z \leq 1{,}000{,}000{,}000$

$x + z \leq 1{,}323{,}440{,}000$

$y + z \leq 2{,}183{,}040{,}000$

$y + z \leq 1{,}800{,}000{,}000$

$x \geq 0, y \geq 0, z \geq 0$

Introducing the slack variable the dual becomes

Maximize

$P' = 2323440X' + 7966080Y' + 19915200Z'$

Subject to:

$z' + r' = 1{,}331{,}920{,}000$

$z' + s' = 2{,}000{,}000{,}000$

$x' + z' + t' = 1{,}000{,}000{,}000$

$x' + z' + u' = 1{,}323{,}440{,}000$

$y' + z' + v' = 2{,}183{,}040{,}000$

$y' + z' + w' = 1{,}800{,}000{,}000$

$x' \geq 0, y' \geq 0, z' \geq 0.$

Tableau 6

	x'	y'	z'	r'	s'	t'	u'	v'	w'	RHS
r'	0	0	1	1	0	0	0	0	0	1,331,920,000
s'	0	0	1	0	1	0	0	0	0	2,000,000,000
t'	1	0	1	0	0	1	0	0	0	1,000,000,000
u'	1	0	1	0	0	0	1	0	0	1,323,440,000
v'	0	1	1	0	0	0	0	1	0	2,183,040,000
w'	0	[1]	1	0	0	0	0	0	1	1,800,000,000
p'	2323440	7966080	19915200	0	0	0	0	0	0	0

Tableau 7

	x'	w'	z'	r'	s'	t'	u'	v'	w'	RHS
r'	0	0	1	1	0	0	0	0	0	1,331,920,000
s'	0	0	1	0	1	0	0	0	0	2,000,000,000
t'	1	0	[1]	0	0	1	0	0	0	1,000,000,000
u'	1	0	1	0	0	0	1	0	0	1,323,440,000
v'	0	0	0	0	0	0	0	1	-1	383,040,000
y'	0	1	1	0	0	0	0	0	1	1,800,000,000
p'	2323440	0	11949120	0	0	0	0	0	-7966080	-14338944 x 10^9

Tableau 8

	x'	w'	t'	r'	s'	t'	u'	v'	w'	RHS
r'	0	0	0	1	0	0	0	0	0	-866,808,000
s'	0	0	0	0	1	0	0	0	0	1,000,000,000
z'	1	0	1	0	0	1	0	0	0	1,000,000,000
u'	1	0	0	0	0	0	1	0	0	323,440,000
v'	0	0	0	0	0	0	0	1	-1	383,040,000
y'	-1	1	0	0	0	-1	0	0	1	800,000,000
p'	-9625680	0	0	0	0	-11949120	0	0	-7966080	-26288064 x 10^9

From Tableau 3′ (the final tableau),

$y' = 800{,}000{,}000$

$z' = 1{,}000{,}000{,}000$

$p' = 26288064 \times 10^9$

For the primal problem

$x_{21} = 11949120$

$x_{32} = 7966080$

$Z' = 26288064 \times 10^9$

Comparing the values of Z and Z'showsome reduction in the total unsecured loan safe to be given out as there is a reduction in the percentage of the constraints.

Now considering a model with mixed constraints;

Minimize

$z'' = 133{,}192{,}000x'_{11} + 2{,}000{,}000{,}000x'_{12} + 1{,}000{,}000{,}000x'_{21} + 1323440000x'_{22}$

$+218304000x'_{31} + 1800000000x'_{32}$

Subject to:

$x'_{21} + x'_{22} \geq 1\% \, of \, 2323440000$

$x'_{31} + x'_{32} \leq 2\% \, of \, 3983040000$

$x'_{11} + x'_{12} + x'_{21} + x'_{22} + x'_{31} + x'_{32} \geq 3\% \, of \, 6638400000$

$x'_{11} \geq 0, x'_{12} \geq 0, x'_{21} \geq 0, x'_{22} \geq 0, x'_{31} \geq 0, x'_{32} \geq 0$

Writing the constraints of its dual in standard form we introduce slack variables as follows:

Maximize $p'' = 232344000x'' + 79660800y'' + 199152000z''$

Subject to:

$z'' + r'' = 133192000$

$z'' + s'' = 2000000000$

$x'' + z'' + t'' = 1000000000$

$x'' + z'' + u'' = 1323440000$

$y'' + z'' + v'' = 2183040000$

$y'' + z'' + w'' = 1800000000$

$z'' \geq 0, y'' \geq 0, x'' \geq 0$

Tableau 9

	x''	y''	z''	r''	s''	t''	u''	v''	w''	RHS
r''	0	0	1	1	0	0	0	0	0	133192000
s''	0	0	1	0	1	0	0	0	0	2000000000
t''	1	0	1	0	0	1	0	0	0	1000000000
u''	1	0	1	0	0	0	1	0	0	1323440000
v''	0	-1	1	0	0	0	0	1	0	2183040000
w''	0	-1	1	0	0	0	0	0	1	2183040000
p''	232344000	-79660800	199152000	0	0	0	0	0	0	0

Tableau 10

	t''	y''	z''	r''	s''	t''	u''	v''	w''	RHS
r''	0	0	1	1	0	0	0	0	0	133192000
s''	0	0	1	0	1	0	0	0	0	2000000000
x''	1	0	1	0	0	1	0	0	0	1000000000
u''	0	0	0	0	0	-1	1	0	0	323440000
v''	0	-1	1	0	0	0	0	1	0	2183040000
w''	0	-1	1	0	0	0	0	0	1	2183040000
p''	0	-79,660,800	-33,192,000	0	0	-2323,440,000	0	0	0	-232344×10^{12}

From the tableau above we have,

$x'' = 1000{,}000{,}000$

$P'' = 232{,}344 \times 10^{12}$

For the primal problem

$x'_{21} = 232344000$

$Z'' = 232344 \times 10^{12}$

Notice that substituting x'' and x''_{21} into the objective function of the dual and primal problems respectively give the same result (232344×10^{12}).

The table below shows the effect of the changes in the parameters on the optimal solutions.

Table 11: Showing the Effect of Unsecured Loan on the Optimal Solution

S/N	Unsecured Medium Term Loan	Unsecured Short Term Loan	Unsecured Total Loan	Optimal Solution
1	≥50%	≥70%	≥20%	$618{,}035.04 \text{ X } 10^{13}$
2	≥0.1%	≥0.2%	≥0.3%	$262{,}880.64 \text{X } 10^{11}$
3	≥1%	≤2%	≥3%	$232{,}344 \text{ X} 10^{12}$

RESULTS AND DISCUSSIONS

From the table above, it is clearly seen that reduction in the unsecured loans reduce the optimal solution. Likewise, the minimization objective function solution is better where the percentage of unsecure loans is reduced. Also, we can see from the table that reducing the unsecured short term loan to*2%*, that is the class of loan which originally was to be greater or equal to *70%* was reduced to less than *2%*, we have optimal solution greater that when this class of loan was ≥0.2%.This implies that the policy of the bank which says that unsecured short term loan should be less than *2%,* as against greater than *0.2%*, does improve the optimal solution. It is important to note that because it is a short term loan, it is not sensitive as regards minimizing the objective function. Finally, the generosity of the bank as regards giving out loan without securities is not without a price. The price they have to pay is that a huge amount is required to be given in order for the banks goals to be achieved, as shown by the optimal solutions. In order to reduce the risk of bad debt incidence to the barest minimum, the banks should reduce the percentage of the unsecured loans since that will not significantly affect the achievement of their aims and objectives.

CONCLUSIONS

Interest is on the loan portfolio management of banks. The analysis is carried out using simplex method and specifically useful for banks whose corporate policy include giving out some percentage of some category of loans without collateral. The peculiar situation is modelled as a linear programming problem. The dual of the minimization linear programming problem is formulated and the resulting maximization linear programming problem is solved using simplex method. A sensitivity analysis is carried out by altering the percentages of the unsecured loans. It is shown that a reduction in the percentage of unsecured loan improves the banks objectives marginally especially when the loan is of a longer term. Also for the bank to be seen as a small and medium scale business friendly bank, the price it has to pay is minimal improvement in her returns from her loan portfolio. It was assumed that the beneficiaries of this unsecured loans are responsible enough to pay back their loans as at when due.

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